Testing for Tau-equivalence and Parallelism Using M*plus*

In this document I explain how to use M*plus* to obtain models based on the assumptions of tau-equivalence, essential tau-equivalence (true-score equivalence), and parallelism. Tests of these assumptions can be obtained by comparing the chi-square values of the models using chi-square difference tests, as explained in the text (pp. 394-395).

Note that the usual CFA model is based on a congeneric model in which item loadings, means, and measurement error variances are all allowed to vary across items. Because of this, no additional model specifications are needed to obtain the congeneric model. Values for that model can be obtained from the document "Confirmatory Factor Analysis Using M*plus*."

The data from this study are the same as those used in the CFA document and consist responses of 1022 people to items on the Achievement Goal Questionnaire (Elliot & McGregor, 2001) collected as part of a study by Finney, Pieper, and Barron (2004). There are no missing data, but I have included the "missing" command used to specify missing data in *Mplus* to demonstrate how this would be done.

The response scale ranged from 1 = not at all true of me to 7 = very true of me. The data are in tab-delimited format and can be found in the file "goal orientation.dat"

Tau-equivalent Model Syntax

The syntax is based on that for the confirmatory factor analysis and repeats the commands for that analysis. I include all commands below but only comment on those that are new to these analyses.

M*plus* commands are in boldface. Non-boldface words indicate information that must be provided by the researcher for their specific data.

I first show the syntax and results for the tau-equivalent model and then show those for the essentially tau-equivalent and parallel models.

Tau-equivalent Model

Title: specification of tau-equivalent and parallel models for goal orientation data;
Data: **file is** goal2.dat;
format is free;
Variable: **names are** i1 i2 i3 i4 i5 i6 i7 i8 i9 i10 i11 i12;
missing all (8,9,10);
usevariables = i7 i8 i9;

! The first set of commands below specifies that item loadings must be equal within factors. The numbers in parentheses at the end of each "by" statement indicate that the values of all parameter estimates specified by that statement must be equal in

value. For example, the statement "perfapp **by** *i1* i2 i3 (1);" indicates that the loadings for i1, i2, and i3 should be equal. Using a different number for each set of "by" statements (here, 1, 2, 3, 4) is indicates that the loadings for each "by" statement must be equal, but that the sets of loadings can differ in value across "by" statements. For example, loading estimates for i1, i2, and i3 must be equal, and estimates for i4, i5, and i6 must be equal, but estimates can differ across the two sets.*

! Inclusion of asterisks (*) after the names of the first item in each line indicate that the item loading should be estimated. Recall that the default in Mplus is to set the loading of the first indicator to 1.0. Use of the asterisk overrides this default.

Model: perfapp **by** i1* i2 i3 (1);
peravoid **by** i4* i5 i6 (2);
masavoid **by** i7* i8 i9 (3);
masapp **by** i10* i11 i12 (4);

! The next set of commands specifies that item means must be equal within factors. Note that each set of item names is enclosed within brackets [ ]. In Mplus, enclosing item names in brackets indicates that item means (or intercepts) should be estimated. Inclusion of the sets of commands for both equality of item loadings and item means results in a tau-equivalent model. As with the loading commands above, the numbers in parentheses are used to specify that estimates of item means must be equal within each factor.

[i1 i2 i3] (5);
[i4 i5 i6] (6);
[i7 i8 i9] (7);
[i10 i11 i12] (8);

! The last set of commands below specifies that items' measurement error variances must be equal within factors. The exclamation points (!) before each command causes Mplus to ignore these, as equality of measurement error variances is not assumed for tau-equivalent models As before, the numbers in parentheses following each command are used to indicate that estimates for each set of measurement error variances must be equal.

! i1 i2 i3 (9);
! i4 i5 i6 (10);
! i7 i8 i9 (11);
! i10 i11 i12 (12);

perfapp@1; peravoid@1; masavoid@1; masapp@1;

! *The command line above sets the variance of each factor equal to 1.0. This specification must be included to identify the model because all loadings are estimated.*

The italicized lines in the syntax above are comments that explain how different models are obtained in Mplus. Comments can be included by preceding a line with an explanation mark (!). Any syntax after the explanation mark will appear in green within Mplus and will not be read by the program. This feature is useful in analyses such as these, in which the different models are invoked by the inclusion or exclusion of particular lines in the syntax.

To obtain results from a tau-equivalent model, I included the commands for equality of item loadings and of item means shown above. I commented out (using explanation points) the syntax for equality of the items' measurement error variances.

These specifications resulted in the output below. I have omitted the factor correlations as they are not relevant to these analyses.

```
Chi-Square Test of Model Fit

          Value                               1371.051
          Degrees of Freedom                  64
          P-Value                             0.0000

MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PERFAPP  BY
    I1                 1.289      0.033     39.257      0.000
    I2                 1.289      0.033     39.257      0.000
    I3                 1.289      0.033     39.257      0.000

 PERAVOID BY
    I4                 1.114      0.039     28.718      0.000
    I5                 1.114      0.039     28.718      0.000
    I6                 1.114      0.039     28.718      0.000

 MASAVOID BY
    I7                 1.109      0.037     30.090      0.000
    I8                 1.109      0.037     30.090      0.000
    I9                 1.109      0.037     30.090      0.000

 MASAPP   BY
    I10                0.797      0.023     33.995      0.000
    I11                0.797      0.023     33.995      0.000
    I12                0.797      0.023     33.995      0.000
```

As can be seen from the output above, the loading values have been set to equality with each factor, as specified in the syntax.

```
 Intercepts
    I1                 5.094      0.043    117.210      0.000
    I2                 5.094      0.043    117.210      0.000
    I3                 5.094      0.043    117.210      0.000
    I4                 4.546      0.044    102.854      0.000
    I5                 4.546      0.044    102.854      0.000
    I6                 4.546      0.044    102.854      0.000
    I7                 4.221      0.042     99.906      0.000
    I8                 4.221      0.042     99.906      0.000
    I9                 4.221      0.042     99.906      0.000
    I10                6.015      0.030    201.070      0.000
    I11                6.015      0.030    201.070      0.000
    I12                6.015      0.030    201.070      0.000
```

The intercept values above are the item means. These have also been set to equality with each factor, as specified in the syntax.

```
 Variances
    PERFAPP            1.000      0.000    999.000    999.000
    PERAVOID           1.000      0.000    999.000    999.000
    MASAVOID           1.000      0.000    999.000    999.000
    MASAPP             1.000      0.000    999.000    999.000
```

The factor variances have been set to 1.0 to identify the model.

```
Residual Variances
    I1                 0.787      0.048     16.348      0.000
    I2                 0.665      0.044     15.091      0.000
    I3                 0.755      0.047     16.001      0.000
    I4                 2.572      0.136     18.885      0.000
    I5                 1.824      0.102     17.801      0.000
    I6                 1.656      0.098     16.888      0.000
    I7                 2.483      0.135     18.336      0.000
    I8                 0.821      0.064     12.762      0.000
    I9                 1.315      0.081     16.333      0.000
    I10                0.812      0.045     17.998      0.000
    I11                0.398      0.030     13.371      0.000
    I12                0.743      0.042     17.841      0.000
```

For the tau-equivalent model, the measurement error variances above are not required to be equal. As can be seen from the values above, they are not.

Essentially Tau-equivalent (true-score) Model

The essentially tau-equivalent model is obtained by dropping the requirement that item means must be equal. You can do this in two ways. The first is to simply omit the commands for item means. Because the default in M*plus* is to include item means and to allow them to vary across items, there is no real need to include these commands unless you want to constrain them in some way.

The other way to do this is to remove the numbers in parentheses (5, 6, 7, and 8) from the item mean commands in the previous tau-equivalent model syntax. This will reduce the commands for item means to:

```
[i1 i2 i3];
[i4 i5 i6];
[i7 i8 i9];
[i10 i11 i12];
```

All other commands remain the same as those for the tau-equivalent model.

Making the change for item means results in the output below:

```
Chi-Square Test of Model Fit

          Value                               440.638
          Degrees of Freedom                       56
          P-Value                              0.0000


MODEL RESULTS

                                                   Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PERFAPP  BY
    I1                 1.293      0.033     39.456      0.000
    I2                 1.293      0.033     39.456      0.000
    I3                 1.293      0.033     39.456      0.000

 PERAVOID BY
    I4                 1.139      0.038     29.814      0.000
    I5                 1.139      0.038     29.814      0.000
    I6                 1.139      0.038     29.814      0.000
```

```
 MASAVOID BY
    I7                 1.067      0.034     31.212      0.000
    I8                 1.067      0.034     31.212      0.000
    I9                 1.067      0.034     31.212      0.000

 MASAPP   BY
    I10                0.813      0.023     35.449      0.000
    I11                0.813      0.023     35.449      0.000
    I12                0.813      0.023     35.449      0.000

 PERAVOID WITH
    PERFAPP            0.504      0.033     15.124      0.000

 MASAVOID WITH
    PERFAPP            0.219      0.037      5.868      0.000
    PERAVOID           0.574      0.035     16.304      0.000

 MASAPP   WITH
    PERFAPP            0.301      0.035      8.655      0.000
    PERAVOID           0.076      0.042      1.794      0.073
    MASAVOID           0.293      0.038      7.690      0.000

 Intercepts
    I1                 5.155      0.049    105.074      0.000
    I2                 5.201      0.048    109.117      0.000
    I3                 4.915      0.048    102.108      0.000
    I4                 4.860      0.060     80.545      0.000
    I5                 4.713      0.055     85.676      0.000
    I6                 4.192      0.052     80.937      0.000
    I7                 5.103      0.050    101.127      0.000
    I8                 4.135      0.045     92.379      0.000
    I9                 3.890      0.048     80.711      0.000
    I10                5.756      0.037    155.984      0.000
    I11                6.209      0.031    201.538      0.000
    I12                5.889      0.037    159.474      0.000

 Variances
    PERFAPP            1.000      0.000    999.000    999.000
    PERAVOID           1.000      0.000    999.000    999.000
    MASAVOID           1.000      0.000    999.000    999.000
    MASAPP             1.000      0.000    999.000    999.000

 Residual Variances
    I1                 0.788      0.048     16.539      0.000
    I2                 0.650      0.043     15.175      0.000
    I3                 0.697      0.044     15.745      0.000
    I4                 2.424      0.128     18.941      0.000
```

```
    I5                    1.796      0.099     18.047       0.000
    I6                    1.444      0.087     16.608       0.000
    I7                    1.463      0.083     17.522       0.000
    I8                    0.908      0.062     14.556       0.000
    I9                    1.235      0.074     16.734       0.000
    I10                   0.732      0.040     18.278       0.000
    I11                   0.310      0.025     12.578       0.000
    I12                   0.734      0.040     18.382       0.000
```

As can be seen from the parameter estimates above, item loadings and measurement error variances, but not item means (intercepts) are now held equal within each factor. Removal of the constraints on item means resulted in a gain of 8 degrees of freedom (2 for each factor) and a decrease of the chi-square value to 440.638.

Parallel Model

Syntax for the parallel model is the same as that for the tau-equivalent model *except* that the exclamation points before the commands for measurement error variances are removed, thus forcing these parameters to be equal within each factor. To obtain the parallel model, the four command lines below are thus added to the syntax for the tau-equivalent model.

```
i1 i2 i3 (9);
i4 i5 i6 (10);
i7 i8 i9 (11);
i10 i11 i12 (12);
```

Adding in these commands results in the output below:

```
Chi-Square Test of Model Fit

          Value                             1582.008
          Degrees of Freedom                      72
          P-Value                             0.0000

MODEL RESULTS
                                                           Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 PERFAPP  BY
    I1                    1.289      0.033     39.245       0.000
    I2                    1.289      0.033     39.245       0.000
    I3                    1.289      0.033     39.245       0.000
```

```
 PERAVOID BY
    I4                 1.126      0.039     28.853      0.000
    I5                 1.126      0.039     28.853      0.000
    I6                 1.126      0.039     28.853      0.000

 MASAVOID BY
    I7                 0.957      0.035     27.357      0.000
    I8                 0.957      0.035     27.357      0.000
    I9                 0.957      0.035     27.357      0.000

 MASAPP   BY
    I10                0.806      0.024     33.449      0.000
    I11                0.806      0.024     33.449      0.000
    I12                0.806      0.024     33.449      0.000

 Intercepts
    I1                 5.090      0.043    117.857      0.000
    I2                 5.090      0.043    117.857      0.000
    I3                 5.090      0.043    117.857      0.000
    I4                 4.588      0.043    105.605      0.000
    I5                 4.588      0.043    105.605      0.000
    I6                 4.588      0.043    105.605      0.000
    I7                 4.376      0.038    115.602      0.000
    I8                 4.376      0.038    115.602      0.000
    I9                 4.376      0.038    115.602      0.000
    I10                5.952      0.029    204.652      0.000
    I11                5.952      0.029    204.652      0.000
    I12                5.952      0.029    204.652      0.000

 Variances
    PERFAPP            1.000      0.000    999.000    999.000
    PERAVOID           1.000      0.000    999.000    999.000
    MASAVOID           1.000      0.000    999.000    999.000
    MASAPP             1.000      0.000    999.000    999.000

 Residual Variances
    I1                 0.734      0.023     31.968      0.000
    I2                 0.734      0.023     31.968      0.000
    I3                 0.734      0.023     31.968      0.000
    I4                 1.987      0.062     31.970      0.000
    I5                 1.987      0.062     31.970      0.000
    I6                 1.987      0.062     31.970      0.000
    I7                 1.644      0.051     31.969      0.000
    I8                 1.644      0.051     31.969      0.000
    I9                 1.644      0.051     31.969      0.000
    I10                0.645      0.020     31.969      0.000
    I11                0.645      0.020     31.969      0.000
```

```
I12                    0.645      0.020     31.969      0.000
```

As can be seen from the parameter estimates above, item loadings, means, and measurement error variances are now held equal within each factor. The additional constraints on the measurement error variances resulted in an increase of the chi-square value to 1582.008.

The significance of the chi-square increases obtained from comparing the congeneric to the essentially tau-equivalent model, the essentially tau-equivalent to tau-equivalent model, and the tau-equivalent to the parallel model are typically summarized in a table such as that shown below. (Results for the congeneric model were obtained from the original CFA output).

Model	χ^2	***df***	$\Delta\ \chi^2$ ***(df)***
Congeneric	283.98	48	
Essentially tau-equivalent	440.64	56	156.66* (8)
Tau-equivalent	1371.05	64	930.41* (8)
Parallel	1582.01	72	210.96* (8)

Addition of the constraints imposed by the essentially tau-equivalent model result in a statistically significant increase of 156.66, indicating that the data deviate significantly from essential tau-equivalence. Alternatively, we could interpret this as meaning that the item loadings within a factor are not equal. Addition of the constraint imposed by the tau-equivalent model that item means must be equal results in a chi-square increase of 930.41.Thus, the Imposition of constraints on the item means yields a statistically significant decrement in fit, indicating that the item means are not equal within a factor.

Finally, addition of the constraints imposed by the parallel model to those imposed by the tau-equivalent model also results in a statistically significant increase in chi-square (210.96 with 8 degrees of freedom). This indicates that the data deviate significantly from the parallel model, which is not surprising because parallelism cannot hold if tau-equivalence does not. This is also a test of the equality of measurement error variances, and indicates that these are not equal across sets of items.